SURREY
MATHS SCHOOL

SuMS PROJECT QUALIFICATION

Project Briefs

Principles of the SPQ

The SPQ is a programme in which our passionate and ambitious students complete collaborative research projects. Guided by professionals in industry or by academics at universities, they will work in groups of three to five on a STEM related problem. The opportunity to complete a meaningful research project relating to these subject areas will help them to develop necessary practical skills that they will need in higher education, and indeed their careers, and will give them a chance to extend their interests beyond the curriculum.

The main outcomes of the project are:

- To develop students' academic literacy.

- To provide students with a direct experience of research.

- To develop students' skills of collaboration, communication and self-management.

- To provide students with the experience of working with either an academic at a university or an industry professional.

Project outcomes

The SPQ project is made up of three parts:

- A written literature review to introduce the problem and to showcase their research skills.

- A written report, poster designed in LaTeX, a podcast or a documentary.

- A five to ten-minute presentation at the end of the school year to staff, parents, governors, mentors and other invited guests.

Acknowledgements

The SPQ project could not have gone ahead without the incredible support of the Faculty of Engineering and Physical Sciences at the University of Surrey. The projects the academics have provided and time they have committed to work with our students is hugely appreciated. Special thanks must go to Maths Education Committee who have coordinated the collaboration between the University and SuMS from the beginning.

We have a number of industry professionals to thank also. Their contributions have further increased the breath of the type of projects offered and have given the students the opportunity to apply the content they study to real-life applications.

Project 1: Object Identification and Decision Making in Driverless Cars

Supervisor: Alexia, Beale, University of Grenoble, France



Project Context

Driverless cars have to rely on image recognition software to identify objects in their field of view. How does an autonomous car decide which objects are hazardous, and if a hazard is perceived, how should we program the decision-making when there is a choice of endangering either the hazard or the car's occupants? In this project, you will explore the technology used for object identification and investigate the ethics behind the decision making of driverless cars.

In the past decade, the ability of image recognition software has advanced rapidly, reaching the point at which driverless cars are no longer a dream. As the technique of object recognition continues to progress, this raises the question of how detailed and accurate object recognition needs to be before driverless cars can be just as "safe" as human drivers. The Moral Machine, **www.moralmachine.net** , explores scenarios where the driver of a car has to make a moral decision. This project explores the approaches to translate ethical decision-making into a mathematical process to program a driverless car.

Guidance for your investigation tasks and research questions

For this project, I suggest you start by researching the current ability of object recognition software in autonomous vehicles, before exploring the ethical

considerations. This is a wide-ranging topic so you might want to choose one research question to focus on. Here are some suggestions:

Object Recognition:

- Which detectors and computational tools does a driverless car use to identify objects?
- What is the current state of self-driving object recognition technology, and what challenges are there for successful object recognition?

Decision-Making:

- If object recognition could be perfected, which parameters should be used in ethical decision-making? And should any parameters be ignored?
- You could extend your research by carrying out a small-scale investigation amongst your peers to discover the human perspective of ethical decisions by asking them to choose between scenarios.
- Alternatively, you could explore computational tools for object recognition by writing a computer code (https://www.geeksforgeeks.org/python/detect-an-object-with-opencv python/)

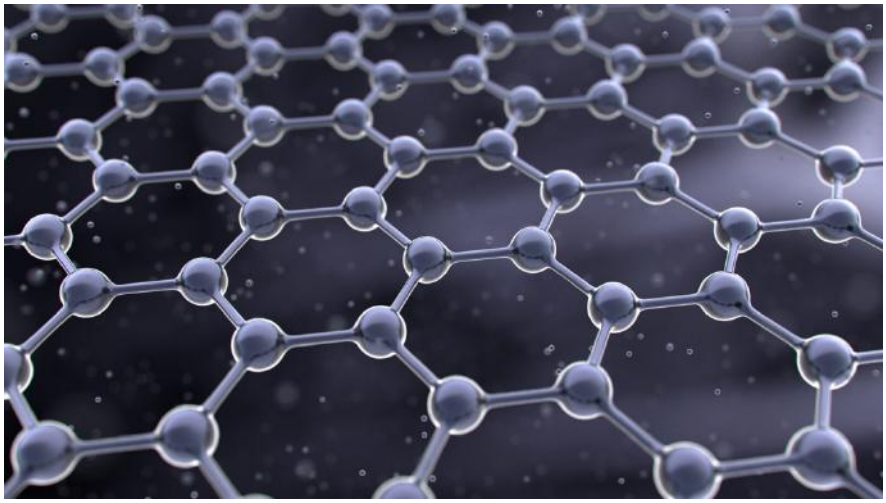
Project output

Your project output should chiefly comprise a literature review and presentation. You should include an overview of the current ability of image recognition software in autonomous vehicles before focusing on an aspect of this topic. A summary of a small-scale investigation of your own choosing would be a valuable addition to the project output.

References: Application and design of a decision-making model in ethical dilemma for self-driving cars, G. Liu (2025), Nature, https://www.nature.com/articles/s41598-025-91921-0

Project 2: One Atom Thick: Growing and Characterising CVD Graphene

Supervisor: Dr Alifah Rahman, School of Engineering, University of Surrey



Introduction

Graphene — a single atomic layer of carbon atoms arranged in a honeycomb lattice is one of the most remarkable materials ever discovered: stronger than steel, thinner than a single atom, and highly electrically conductive. Yet graphene does not exist in nature in a usable form. It must be grown deliberately, using a process called Chemical Vapour Deposition (CVD), and the quality of the result depends critically on the choices made during growth.

In CVD, methane gas (CH_4) is passed over a heated metal substrate, most commonly copper foil at around 1000°C. The metal acts as a catalyst, lowering the energy needed to break apart the methane molecules and freeing carbon atoms that self-assemble into a graphene sheet on the surface. A central challenge is controlling whether the result is a single continuous monolayer or a thicker, lower-quality multilayer film. The choice of substrate material, its surface preparation, and the growth temperature and gas pressure all play a role.

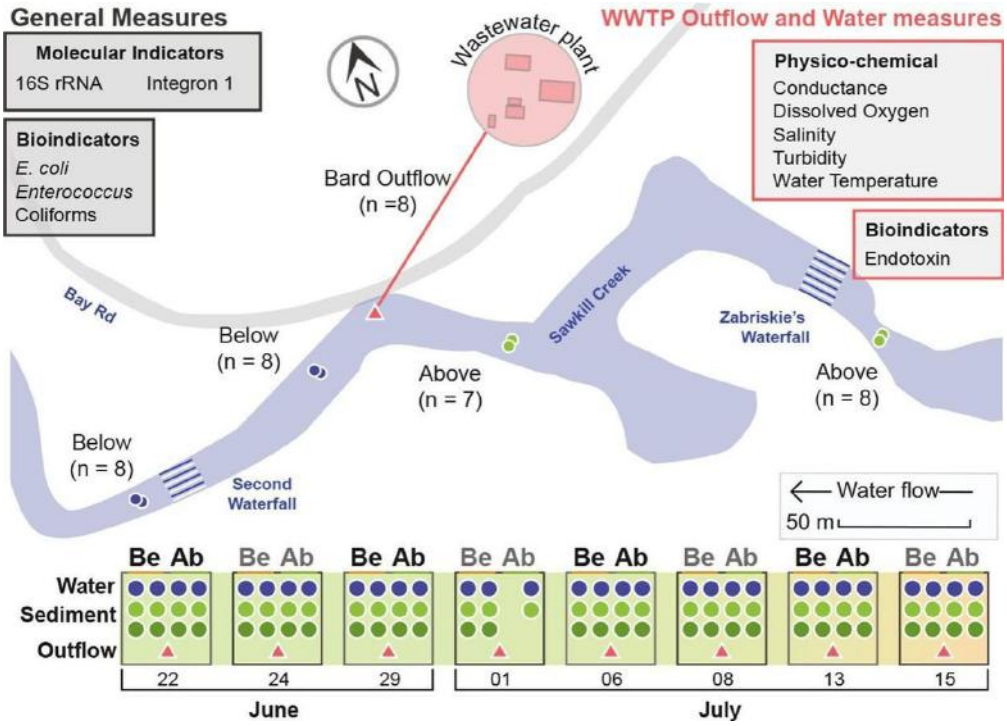
How do researchers know what they have grown? The answer lies in Raman spectroscopy — a technique in which a laser is shone onto the graphene surface and the scattered light reveals a fingerprint of peaks whose relative heights and positions identify whether the sample is a true monolayer, a bilayer, or a thicker multilayer. This project investigates the science behind CVD graphene synthesis, the role of the substrate, and the use of Raman spectroscopy to characterise and optimise the quality of the grown film.

Problems to be investigated

- Substrate selection for CVD graphene growth: why is copper the preferred catalyst, and how does the substrate material, crystal orientation, and surface preparation affect the nucleation and quality of the graphene layer?
- Growth parameter optimisation: how do temperature, methane concentration, and gas flow rate determine whether CVD produces a continuous monolayer or a patchy, defect rich multilayer film? What conditions favour single-layer growth?
- Layer number and quality from Raman spectroscopy: the origin of the G, D, and 2D peaks in graphene's Raman spectrum, why the ratio of the 2D to G peak intensity distinguishes a monolayer from bilayer and few-layer graphene, and what the D peak reveals about defects.
- Graphene grain boundaries and domain size: what are graphene grains, how do they form during CVD, and how does grain boundary density affect electrical properties? How can Raman mapping reveal the spatial uniformity of a grown film?

Project 3: Monitoring and Modelling Bacteria Levels in the River Wey

Supervisor: Alison Taylor, Mathematics and Physics Department, University of Surrey



Project context

The River Wey is popular for a variety of water sports, including Kayaking, paddleboarding and canoeing. As you may be aware people who enjoy water sports are becoming more concerned about water quality and in particular the presence of e-coli or other harmful bacteria that can lead to illness. Harmful bacteria can find their way into rivers through various mechanisms including sewage discharge and agricultural runoff. Bacterial levels in the River Wey are currently monitored by volunteer testers as part of a Citizen Science project they also monitor the chemical and physical properties of the water at the sample sites. In-situ sampling is one approach to assessing water quality at specific geographical locations at a particular time. However, in order to assess the quality over the wider river system and at different times, modelling may be necessary. This could take the form of a simple interpolation between the sampling sites or a more sophisticated machine learning approach as for example in the paper of Aldein *et al* (2026.)

Guidance for your investigation tasks and research questions

You should start by investigating information in the links above. There are a number of directions your project could take. You do not need to investigate all the possibilities. Some examples are:

- Does there seem to be any connection between the bacterial concentrations and the chemical and physical properties of the water at the sample sites? Visit the Community lab site and review the results on the testing site pages.
- Can you create a 2-dimensional visualisation to represent the bacterial concentrations in their geographical locations on the river?

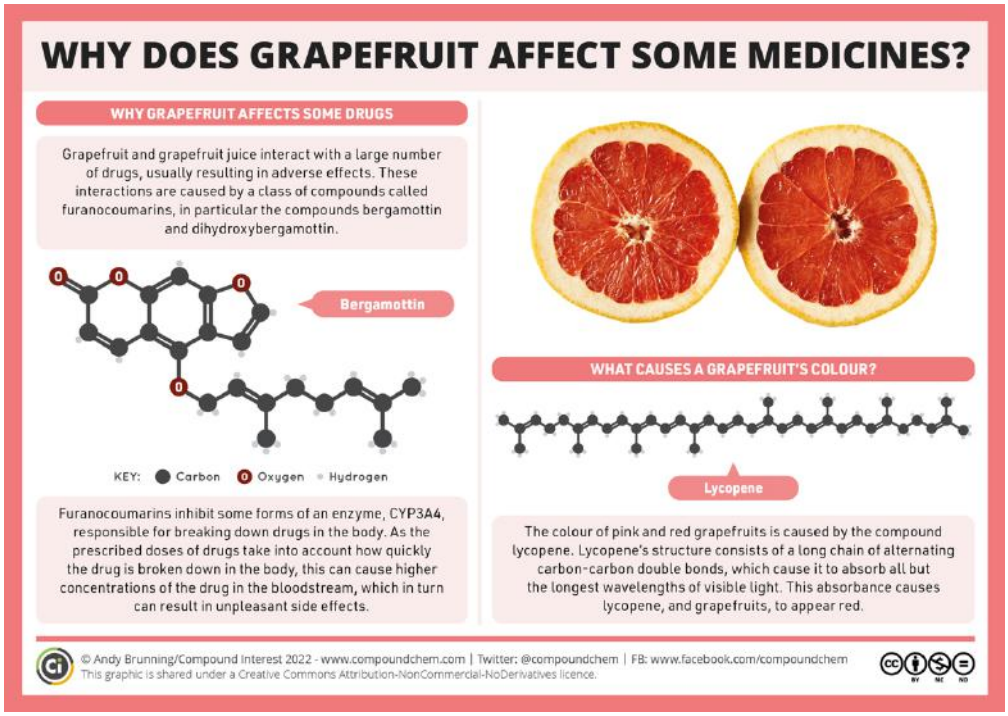
- Investigate how you could estimate the bacterial concentrations by modelling, in areas that are not physically sampled. What additional data might be needed to create such a model?
- Using any available data and a model of your choice can you carry out a small-scale investigation of your own to estimate the bacterial concentrations? If using a machine learning model you could use the data from some sites to train the model and verify the results against the test site data.
- Discuss the merits of different approaches and which approach would you recommend for the River Wey?

Project output

Your project output should comprise a literature review and presentation as well as a summary of the findings of any investigation which your group may have carried out.

Project 4 Grapefruit Juice and Drugs

Supervisor: Carol Spencely, School of Engineering, University of Surrey



Project context

Medicinal drugs are given via different routes; examples include injections, pills that are swallowed, nasal sprays. Drugs are given at different doses for different people – large adults need a different dose to a small baby. However, adults who weigh the same may require different doses for the same drug. Why? How do we calculate drug doses for each person? And why can't we take the flu vaccine as a tablet? Pharmacokinetics help us to explore the answers to these questions.

Pharmacokinetics includes 4 main processes: absorption, distribution, metabolism and excretion (ADME). There are formulae used to calculate ADME of drugs to help clinicians.

There are lots of variables to factor in when considering the pharmacokinetic properties of drugs; for example, food, drink and sometimes other medications can interact with each other. Grapefruit juice is a commonly listed food stuff to avoid when taking certain drugs (e.g. citalopram) – pharmacokinetics helps us to understand why.

Guidance for your investigation tasks and research questions

What would be the result of a patient drinking grapefruit juice and taking citalopram?

- What is pharmacokinetics?
- What are ADME? Explain each.
- What are some of the main formulae involved in calculating the ADME of a drug?
- What is citalopram?

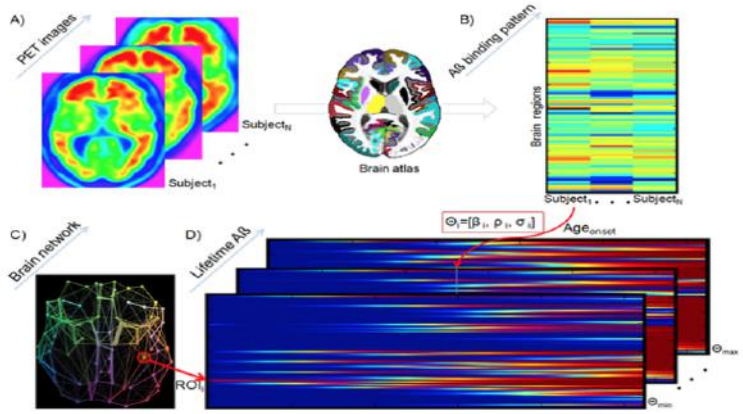
- How does grapefruit juice affect the pharmacokinetics of citalopram? Why?
- Why does grapefruit juice not affect the pharmacokinetics of paracetamol?

Project output

Your project output should comprise a literature review and presentation. Your literature review could cover the questions listed above. Your presentation could use an example of a patient (share their age, weight, BMI and any other relevant information) and give them a dose of drug then, using the appropriate ADME routes and formulae, you can calculate what happens to the drug. Then what happens when they have grapefruit juice for breakfast.

Project 5: Can we Use Maths to Characterise Healthy and Pathological Ageing?

Supervisor: Daniel Abasolo, School of Engineering, University of Surrey



Project context

The use of non-invasive technologies to analyse brain activity is an essential part of research in neuroscience and biomedical engineering. Non-invasive techniques such as functional magnetic resonance imaging (fMRI), computed tomography perfusion (CTP), or positron emission tomography (PET). Moreover, the use of these techniques to investigate the brain has led to the generation of essential information about the effects of ageing on the brain.

Nevertheless, the cost and time-consuming nature fMRI and PET make these tools unavailable to most, so alternative options must be developed. One possible solution involves the analysis of electroencephalogram (EEG) signals, non-invasive recordings of the electrical activity of the brain.

Your investigation task

In this project the goal will be to explore whether mathematical algorithms can be used to extract information from EEG signals that could contribute to the understanding of healthy and pathological ageing. The following tasks might help you getting started:

- What do EEGs measure?

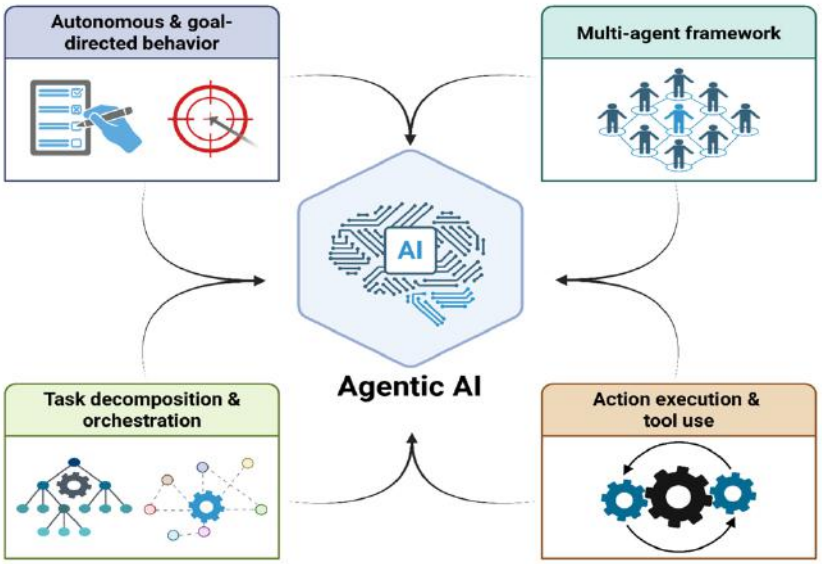
- What type of changes happen to the brain during maturation and ageing? Can those changes be captured in EEG signals?
- Are changes in the brain with healthy ageing like those happening because of dementia or other neurodegenerative diseases?

Project output

Students are expected to complete a thorough literature review of mathematical algorithms applied to the analysis of EEGs. Based on this work, you will need to propose an analysis pipeline that could potentially be applied to the characterisation of healthy ageing using EEG signals, and the early diagnosis of Alzheimer's Disease and/or other forms of dementia, identifying the advantages and disadvantages of the different methods, and discussing the potential role of artificial intelligence in this task.

Project 6: Solving Real-world Problems with Multidisciplinary AI Agents

Supervisor: Dominic Mahon, School of Education, University of Surrey



Project context

Usually, real world problems that arise in engineering, technology, healthcare and many other areas are approached by multidisciplinary teams. These teams comprise experts in different discipline areas since the problems are too complicated to be solved by experts in one particular domain. For example, redesigning a city centre to reduce carbon emissions might require a team of urban planners, civil engineers, environmental scientists, economists, and sociologists. In this interesting project, imagine yourself as the team leader, responsible for solving a problem that can be in one of the areas above, or another of your choosing. However, instead of managing a human team, you are managing a team of expert AI agents representing different disciplinary perspectives, aligned with the problem scenario you have chosen. Each AI agent knows different content, but they also approach the problem differently, asking different questions, privileging different forms of evidence, identifying different risks and sometimes reaching conflicting recommendations. Your role would be to question, coordinate and synthesise those different perspectives to present a possible solution to the problem you have posed.

Guidance for your investigation tasks and research questions

It is important to focus on a concise starting point. Your project could comprise a purely theoretical investigation, a practical investigation using available AI platforms or some combination of theory and practice.

- Choose a small-scale problem-solving scenario of interest to the group and begin to investigate by reviewing some of the literature on using AI teams (or committees) for solving the problem.
- What are the key challenges of using a team of AI experts?
- How might decisions made by the experts or information provided by them be fused together to reach some form of consensus view on an optimal solution to the problem?

- You may want to try some experiments with a small number of agents (say 3-5) applied to your problem and use this as a case study.

Project output

Your project output should comprise a literature review and presentation as well as a summary of the findings of any investigations or case study which your group may have carried out.

Project 7: STEM Meets Volcanoes: Advancing Climate Awareness Through Hazard Mapping in Indonesia

Supervisor: Fadhli Muhibuddin, Universitas Negeri Malang, Indonesia



Project Context

Indonesia is home to more than 120 active volcanoes, making it one of the most hazard-prone regions in the world. Volcanic hazard maps are critical tools for risk mitigation, spatial planning, and community preparedness for an eruption. Furthermore, in addition to the immediate risks posed to local communities, volcanoes discharge gases such as CO2 and SO2 as well as particulates.

Plumes of gases and particulates reach high into the atmosphere contributing to local and wider spread natural climate change. Residents in countries remote from any volcanic activity may not be aware that these natural geological processes influence climatic conditions. Neither may they realise how local communities and their infrastructure are adapted to the constant threat of disaster from volcanic activity.

This fascinating interdisciplinary project offers you the opportunity to carry out an investigation in the context of the United Nations Sustainable Development Goals (SDGs): SDG 13 (Climate Action) and SDG 4 (Quality Education). You can explore digital mapping and Geographic Information Systems GIS technology applied to local data on the volcanoes and climate in Indonesia as well as disaster engineering and risk modelling. You can also consider how hazard maps can be used as powerful tools to enhance climate literacy and disaster awareness and how they could potentially be integrated within an educational curriculum.

Research Tasks & Questions

- Using available data carry out a small-scale case study investigation into the impact of volcanic activity on a specific aspect of local climatic conditions in Indonesia.
- Which technologies (e.g. VR, AR, GIS-based e-learning) are most effective for visualising volcanic hazard data in an interactive environment?
- Provide a recommendation on how your findings from 1-3 could be used to enhance climate literacy within an educational curriculum.
- To what extent has your project contributed to the achievement of SDG 13 (Climate Action), SDG 4 (Quality Education)?

Expected Output

A report and presentation combining statistical analyses of available data, including an example of an interactive spatial map and recommendations for enhancing climate literacy.

First Steps / Suggested Resources

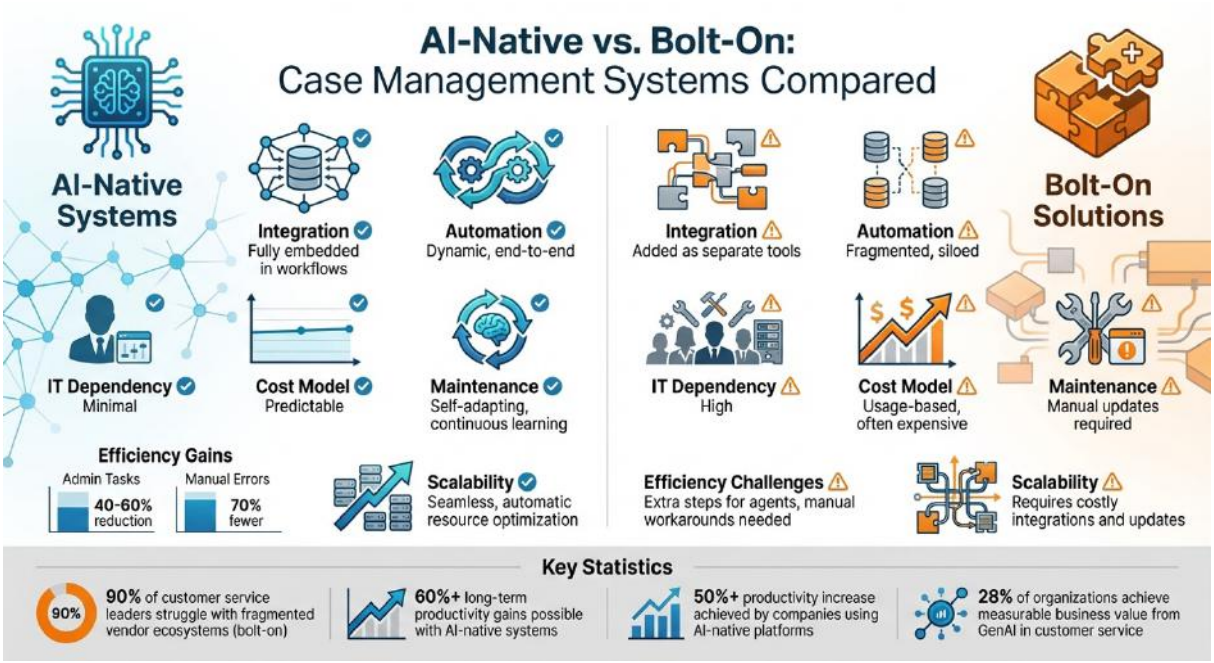
Review literature on volcanoes in Indonesia and the impact that volcanic eruptions can have on climate.

Skills Developed

This is one of two SPQs where you can collaborate with a mentor and a small group of students at a university in Indonesia (Universitas Negeri Malang.) You will develop skills in cross-cultural collaboration, interdisciplinary team discussions, SDG-focused research, applied statistical analysis, using GIS tools and research communication.

Project 8: Helping Shape the Future of an AI Business

Supervisor: Ian Turfley (Technology Transformation Group (TTG))



Project context

In this project, your mentor will be Ian Turfrey, the CEO of [Technology Transformation Group (TTG)]. TTG is a growing technology and AI business with a team of approximately 40 specialists across software engineering, digital transformation, AI and Microsoft technologies. Their challenge is not the technology itself but explaining what they do in a way that is simple, compelling and easy for senior business decision makers to understand. Importantly it should be clear for customers to see how their “Astra” brand differs from traditional consultancies and software vendors. This isn't a theoretical exercise. Your recommendations could directly influence how TTG positions its products, goes to market, and scales its business as they pursue ambitious growth plans over the coming years. You'll be working on a live challenge at the intersection of AI, innovation, marketing, strategy and entrepreneurship.

Guidance for your investigation

- How can the Astra portfolio be structured and presented?
- How can each offering be positioned in the market?
- How should customers navigate the portfolio?
- How can TTG communicate a clear and compelling value proposition?
- How should TTG approach marketing and go-to-market for our key propositions?

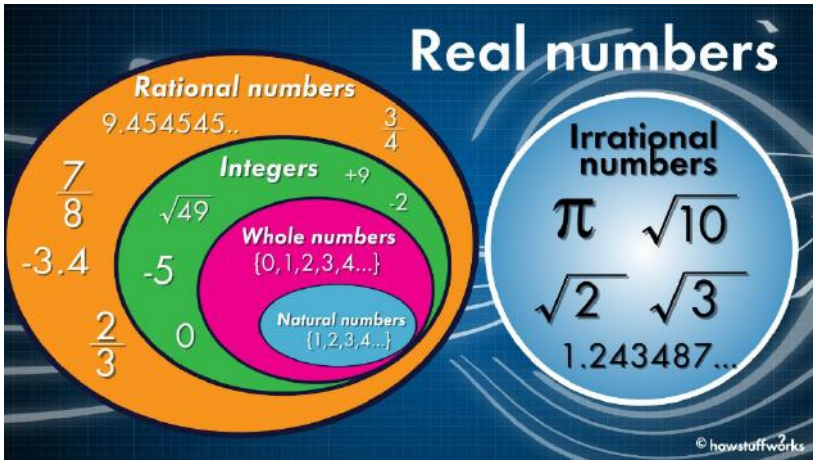
Where to start? Prior to your first meeting with Ian you should visit the TTG website linked above and begin to familiarise yourselves with the company, what they do and their project portfolio.

Project output

Your project output should comprise a literature review and presentation which sets out recommendations based on your investigations, as well as an outline strategy for implementation of your recommendations.

Project 9: Rational and Irrational Numbers

Supervisor: Jonathan Deane, Department of Mathematics, University of Surrey



Project context

This project concentrates on proving some facts about the real numbers and you will study several examples of proof by contradiction. The real numbers, e.g. −0.1, 2π, 0.693 can be visualised as lying on an infinitely long line, known as the real line, tending to −∞ at one end and +∞ at the other. In this project, we look at some aspects of the delicate structure of the real line. Each finite real number can be classified as being either rational or irrational. A number q is rational if we can write q = a/b where a and b are integers (whole numbers) with no common factors and with b/=0. Hence 22/7, −3 and 2/5 are rational — all the integers are rational because, when common factors between numerator and denominator are cancelled, the denominator is 1.

If a number is not rational, then it is irrational. Examples include √2 and the base of natural logarithms, e ≈ 2.7182818.... When we write an irrational number as a decimal, the string of digits goes on for ever, usually with no obvious pattern (although I can show you an example where this is not the case).

We will examine some proofs of simple facts about rational and irrational numbers and we will also see how to prove irrationality for some particular numbers, e.g. √2 and e.

Suggested areas of investigation

A small set of numbers can be written as a list of the members of the set enclosed in { }. For instance, the set of the factors of 6 is {1,2,3,6}. Conventionally, the set of all rational numbers is given the symbol **Q**; the set of the positive integers, {1,2,3,...}, also

known as the natural numbers, is **N**; and the set of all integers, both positive, negative and zero, is written **Z**. We use the symbol ∈ to mean 'is a member of'; for instance, 2/3 ∈ **Q** and −12 ∈ **Z**. The set difference between sets A and B, written A\B, is the set of the elements of A that are not in B, e.g. {1,2,3,4,5} \ {1,4} = {2,3,5}. There is no standard symbol for the set of irrational numbers; instead, we usually write this set as **R** \ **Q**, the real numbers with the rational ones removed.

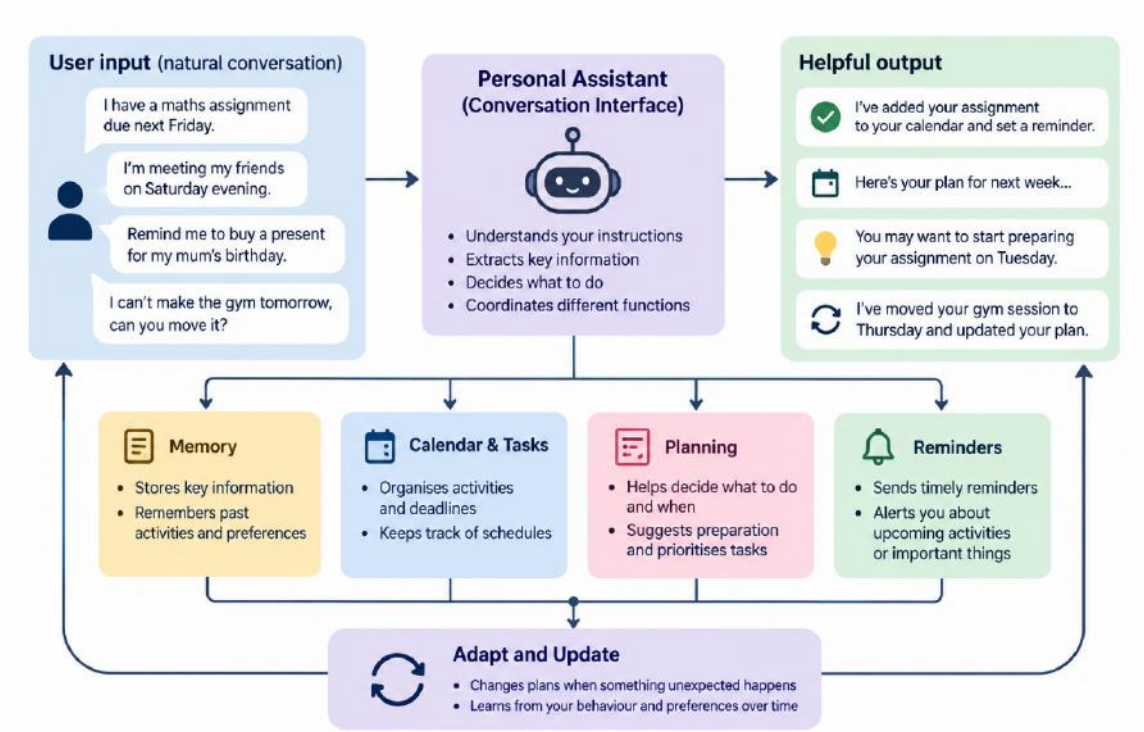
- (1) The following easily-proved results will be useful: if x is irrational and q is rational, then x + q and xq are both irrational. If x and y are both irrational, what can you say about x+y and xy? If q and r are both rational, what can you say about q +r and qr?
- (2) Between every pair of irrational numbers x and y, with x/= y, we shall see how to prove that there lies another irrational number, z say, which is not equal to either x or y. The same is true for every pair of unequal rational numbers. This leads to the delicate structure referred to above.
- (3) We will examine a proof that √2 is irrational — this proof goes back to ancient Greece, approximately 500 BC. Can we generalise the argument?
- (4) We will examine a nice proof, due to Joseph Fourier (1768–1830), that e, which is defined by e := ∞ n=0 1/n!, is irrational. Can we generalise this argument?
- (5) We could look at related matters, for instance, a proof due to Joseph Liouville (1809–1882), that e2 is irrational; and possibly the idea of countability, whereby we can argue that in a mathematical sense there are 'more' irrational numbers than rational numbers. The ideas here are profound and go under the general label 'countability'. Their originator was Georg Cantor (1845–1918).

Aims and outcomes

This project is designed to be open-ended, to give you something to think about and allow you to decide how far you want to explore the ideas. The suggestions above are designed to give you a framework. If at the end of the project you have an increased understanding of any or all of, for instance, (i) what mathematical proof is; (ii) proof by contradiction, with a good understanding of some specific examples; (iii) how to present some of this material to a non-mathematical audience in such a way that they can appreciate the main ideas; then you will have succeeded.

Project 10: Talk, Plan and Organise Your Life

Supervisor: Kai Yang, Computer Science & EEE, University of Surrey



Project context

Every day, we have many things to remember and organise. These may include school or work, meetings, deadlines, family activities, appointments, travel, birthdays and other personal plans. We already use calendars, reminders and to-do lists to help us, but we still need to enter information ourselves, remember what needs to be done and decide when to do it.

Imagine having a personal assistant that you could simply talk to. You could say, “I have an assignment due next Friday”, “I am meeting my friends on Saturday”, or “My mum’s birthday is next month”. The assistant could remember these things, help organise them and remind you at the right time.

But could a personal assistant do more than simply remember? For example, if you have an important deadline next week, could it help you plan when to start preparing? If you suddenly become busy tomorrow, could it rearrange your other activities? Could it notice that an important event is approaching and suggest something you may need to prepare? Over time, could it also learn how you prefer to organise your day and when you do or do not want to be reminded?

Recent AI developments are making this type of personal assistant increasingly possible. An interesting challenge is therefore to investigate how such an assistant could help people organise not only their tasks, but their everyday lives, while still allowing people to remain in control.

Guidance for your investigation tasks and research questions

You may find it easier to start by exploring how people currently use calendars, reminders, digital assistants and other tools to organise their everyday lives.

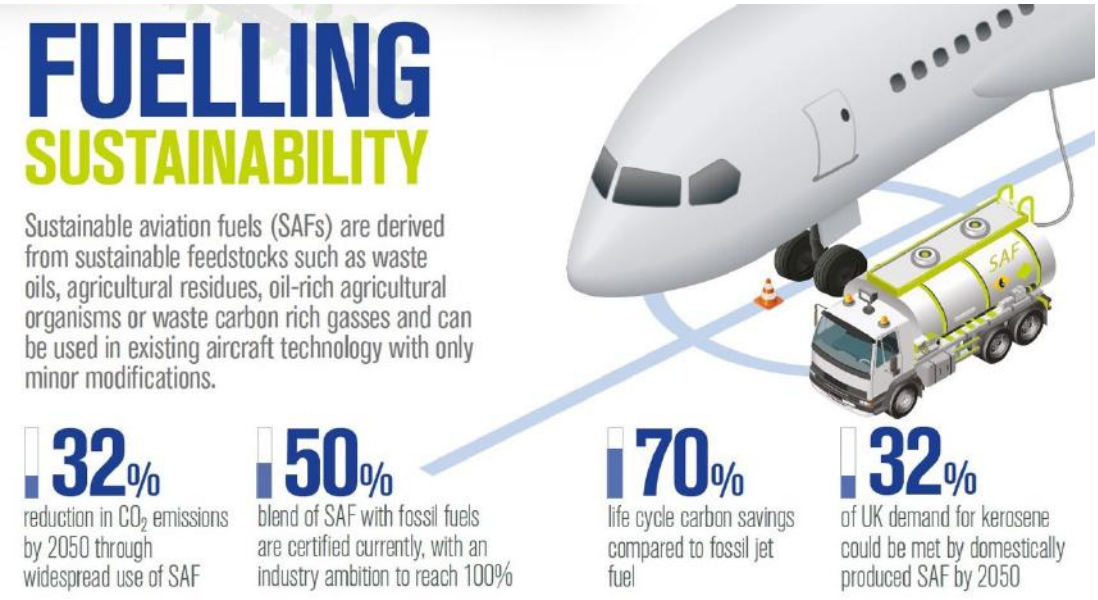
- What should a personal assistant be able to help with, such as remembering activities, managing deadlines, planning ahead, organising everyday life and providing reminders?
- How could different functions such as conversation, memory, calendar, planning and reminders work together as one personal assistant?
- How could the assistant help users decide what to do next, prepare for upcoming activities and identify things they may have forgotten?
- How should the assistant respond when plans change, new activities are added or something is not completed as expected?
- Using available AI tools, can you design and test a simple personal assistant that combines some of these functions in different everyday situations?

Project output

Your project output should comprise a literature review and presentation exploring how a smart personal assistant could help people organise their everyday lives.
You could design and test a simple personal assistant using available AI tools, and explore how well it can support everyday activities such as remembering, planning, reminding and adapting to changes.

Project 11: Will Sustainable Aviation Fuel Take Off?

Supervisor: Mary Hilditch, School of Engineering, University of Surrey



Project context

Sustainable aviation fuel (SAF) is a term used to describe renewable alternatives to kerosene-based aviation fuel such as Jet-A. Several different types and sources of fuel fall under the sustainable category and are considered as SAF.

Despite some high-profile headlines such as Virgin making the first transatlantic airline flight powered by 100% SAF in 2023, SAF is not widely used and powers less than 1% of global flight operations [1]. In Europe and the UK the aviation industry is currently required to blend a minimum of 2% SAF into all aviation fuel sold.[2]. This is mandated to rise to 20% by 2035, and 70% by 2050. However, industry observers are questioning whether this will be achieved.

Some questions to consider

- What are main methods of making SAF? Are new methods underdevelopment?
- How is SAF considered a sustainable fuel? Does it still contribute to CO_2 production?
- What are the barriers to wider use of Sustainable Aviation Fuel?
- Can SAF be created as a “drop-in fuel”? Allowing aircraft engines to switch to burning SAF without engine modifications?
- What are the main things which need to happen for SAF to be more widely used and the UK meet its SAF targets?

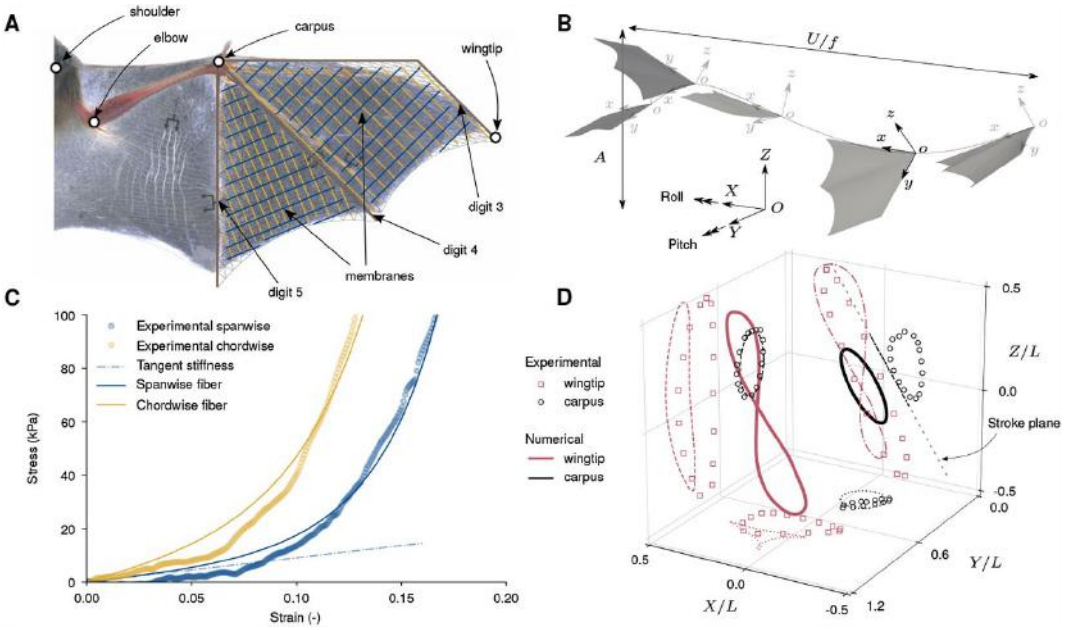
Background references

[1] What Is Sustainable Aviation Fuel (SAF)? The Complete 2026 Guide | AirMedia24

[2] https://www.gov.uk/government/collections/sustainable-aviation-fuel-saf-mandate-statistics

Project 12: Numerical Modelling of Avian Flight Trajectories

Supervisor: Olaf Marxen, School of Engineering, University of Surrey



Project context

Despite the recent proliferation of computation, sensing, and actuation at the small scale, aerial robots are still not able to match the agility and adaptability of natural flight. This is due, in part, to our limited knowledge of the mechanics that characterise outdoor flight. An example of a bird's superior capability is the avian perching manoeuvre, during which a bird decelerates to land on a spot, such as a tree branch. In this project, data characterising bird motion during flight, including perching manoeuvres, will be analysed, and a computational model could be developed.

The rationale behind this project is that we would like to understand, and be able to model, the forces and trajectory of a "bird landing", with a view to enable future fixed-wing unmanned aerial vehicles to perform similar, highly dynamic manoeuvres. In addition to biomimetic flight of aerial robots and a deeper understanding of biological flight behaviours, other applications can be envisioned, such as wind-turbines alleviating gust loads during operation and hence avoiding costly showdowns.

Your Investigation Tasks

Your tasks will be based around analysing qualitative and quantitative data of perching birds, followed by developing a numerical model of the underlying flight dynamics. To familiarize you with the task, you will analyse with video data that we

took of flying birds of prey as well as motion tracking data from flight experiments with artificial flyers that we performed in the wind tunnel. You could use computational tools to deduce velocities and acceleration from trajectory data, and then attempt to develop a numerical model (physics-based or data-driven), e.g. solving a set of ordinary differential equations or applying machine learning techniques, that allows to simulate flight trajectories, using data to inform or validate your model. You should have or be in the process of developing, skills and knowledge in analytical and numerical differentiation and integration and be able to work with (or learn to work with) programming languages such as Python, MATLAB or similar.

Possible outcomes

One project outcome may be, for instance, a dynamical model for visualizing the motion and forces of birds or artificial flyers, and you could demonstrate the use of this model during a presentation. You could also showcase the use of this model and discuss the time-dependent evolution of forces (gravity, drag and lift) and their relative contributions. Your conclusions should highlight features of avian(-inspired) flight deduced from your analysis.

Resources

You will be provided with video data of avian flight, in particular perching, and results from wind-tunnel testing. Depending on time and interest, you may be able to visit the University of Surrey to perform your own experiments. You will get support with numerical techniques for solving differential equations.

How to get started

Begin by studying the literature to understand what is currently known about avian perching manoeuvres (e.g. **https://www.nature.com/articles/s41586-022-04861-4**). Then familiarize yourself with avian perching by looking at available video data.

Project 13: Foundations of Mathematics in Isabelle

Supervisor: Paulo Emilio de Vilhena, Computer Science & EEE, University of Surrey

```
lemma BFs_3[PLM]:
  "[◇(∃ α. φ α) → (∃ α . ◇(φ α)) in v]"
  proof -
    have "[(∀α. □(¬(φ α))) → □(∀α. ¬(φ α)) in v]"
      using BF by metis
    hence 1: "[(¬(□(∀α. ¬(φ α)))) → (¬(∀α. □(¬(φ α)))) in v]"
      using contraposition_1 by simp
    have 2: "[◇(¬(∀α. ¬(φ α))) → (¬(∀α. □(¬(φ α)))) in v]"
      apply (PLM_subst_method "¬□(∀α . ¬(φ α))" "◇(¬(∀α. ¬(φ α)))")
      using KBasic2_2 1 by simp+
    have "[◇(¬(∀α. ¬(φ α))) → (∃ α . ¬(□(¬(φ α)))) in v]"
      apply (PLM_subst_method "¬(∀α. □(¬(φ α)))" "∃ α. ¬(□(¬(φ α)))")
       using cqt_further_2 apply metis
      using 2 by metis
    thus ?thesis
      unfolding exists_def diamond_def by auto
  qed
lemmas "BF◇" = BFs_3
```

Project context

What are the basic rules of mathematics, and can everything be built from them? At the end of the nineteenth century, mathematicians began to ask these questions because paradoxes such as Russell's paradox showed that intuitive ideas about sets could lead to contradictions. Frege, Russell, Hilbert and others tried to place mathematics on firm logical foundations by devising a small collection of axioms and rules of reasoning from which all mathematics could be derived. Today, these foundations have a very practical use in the form of proof assistants. A proof assistant is a computer program where you can write mathematical proofs, concepts, and definitions and have them automatically checked with respect to the logical rules of mathematics. Proof assistants have changed the way mathematicians do mathematics and continue to shape the future of mathematics. This project invites you to discover Isabelle (isabelle.in.tum.de), a proof assistant developed jointly at the University of Cambridge and at the Technical University of Munich (TUM). This experience will teach you how proof assistants work and how mathematics is built from simple logical steps.

Guidance

Our goal will be to formalise a couple of mathematical results, such as the irrationality of the square root of 2, the infinity of primes, or Cantor's theorem, the result that there is no surjective function from N to P(N) (the set of sets of natural numbers).

We will proceed in two steps:

- Write down the proofs of these results in English as explicitly as possible.
- Translate these proofs to Isabelle.

The first step will require learning some key proof methods of mathematics, such as proof by induction and proof by contradiction, as well as getting comfortable with the overall rules of logic. The second step will require learning Isabelle. We will proceed gently by presenting the general concepts of Isabelle (how statements and proofs are written, how numbers are represented, etc.) and by performing the proofs of simple logic statements.

Outcome

The output will be (1) a formal artefact consisting of the formal proof written in Isabelle of the results above (irrationality of √2, infinity of primes, Cantor's theorem, etc.) and (2) a presentation explaining what are proof assistants and how they affect the students' understanding of mathematics.

Project 14: Cumulonimbus

Supervisor: Richard Harrison, Computer Science & EEE, University of Surrey



Project context

Cumulonimbus are spectacular storm clouds that can tower 20km or more vertically. They may span the entire troposphere and even enter the stratosphere. These clouds have a huge mass, typically ranging from ~105 – 106 tonnes. That's the mass of around 100 Eiffel Towers! In extreme cases, vertical windspeeds inside the clouds can exceed 150kmh creating severe turbulence. Individual hailstones formed in the clouds can have masses approaching 1 kg and can coalesce to form ice accretions up to 4kg in mass. Cumulonimbus clouds present a very significant danger to aviation due to extreme forces from wind shear and the potential for hailstone damage. There are several recorded cases, such as NLM Cityhopper 431 where passenger aircraft have been destroyed or severely damaged through an inadvertent encounter with one of these giant clouds. Hailstones can impact the ground at over 150kmh causing extensive damage to buildings, vehicles and crops as well as threatening human and animal life. In the 1888 Moradabad hailstorm there were 246 fatalities, primarily due to severe head injuries caused by the hailstones. Intense electrostatic build up inside the clouds ionises the air transforming it into a plasma which forms highly conductive pathways, visible to us as lightning. Lightning can also be a threat to life, as well as causing serious environmental damage such as wildfires.

Guidance for your investigation tasks and research questions

- Prepare a brief case study which highlights the dangers posed by cumulonimbus clouds. You could choose one of the two examples from above or another of your own choosing (there are plenty to choose from.) This will form part of your literature review.
- Review the basic physics concepts behind the formation of Cumulonimbus clouds and the mathematical models for their formation.
- Hazards from cumulonimbus clouds include hailstones, strong winds and electrical discharge. Hailstone formation inside a cumulonimbus cloud can be modelled by first order, linear ordinary differential equations. You could review some of the literature on hailstone formation then using suitable differential equations and parameters, implement a simple model of hailstone formation using computer code (which you can easily generate with AI.)
- You do not have to focus on hailstone formation, you could instead investigate some of the physical/mathematical models behind the formation of strong winds and electrostatic build up and discharges that occur within cumulonimbus clouds.

Project output

Your project output should comprise a literature review and presentation as well as a summary of the findings of any modelling investigation which your group may have carried out.

Project 15: Linking Health and Learning: Do Nutrition and Well-Being Shape the PISA Mathematics Attainment Gaps between the UK and Indonesia?

Supervisor: Rochamat Aldy, Universitas Tidar, Indonesia



Project Context

The United Nations Sustainable Development Goal 3 (Good Health and Well-Being) and SDG 4 (Quality Education) are closely interconnected. Good nutrition and overall well-being are critical foundations for learning capacity, cognitive performance, and educational attainment. In Indonesia, disparities in diet quality and access to affordable healthy food may be detrimental to students' health and readiness to learn. Indonesia's 15-year-old students have consistently scored below the OECD average in Mathematics on the Programme for International Student Assessment (PISA). The 2022 score for Indonesia was 366 compared to the global average of 472. The score for the UK was 489 but there is a much higher variability in the PISA performance in the UK compared to Indonesia. This raises some important questions:

Research Questions

- To what extent do nutrition and well-being factors (diet quality, food affordability, health status) contribute to explaining the performance gap between the UK and Indonesia?
- Why is there a much higher variability in the PISA performance of UK students compared to Indonesia?

- How do socioeconomic, institutional, and technological factors interact with health and nutrition in shaping mathematics performance in the UK and Indonesia?
- What strategies could integrate health, nutrition, and education policies to reduce mathematics attainment gaps and enhance student outcomes?

Expected Output

A report and presentation combining statistical analyses of available data with cross-disciplinary insights and recommendations for reducing mathematics attainment gaps.

First Steps / Suggested Resource

Review PISA datasets and OECD research for the UK and Indonesia.

Skills Developed

This is one of two SPQs where you can collaborate with a mentor and a small group of students at a university in Indonesia (Universitas Tidar). You will develop skills in cross-cultural collaboration, interdisciplinary team discussions, SDG-focused research, applied statistical analysis and research communication.

Project 16: Where Does Our Food Waste Go?

Supervisor: Siddharth Gadkari, Dept. of Chemical Engineering, University of Surrey



Project context

The UK throws away around ten million tonnes of food a year after it leaves the farm, and roughly 60% of that comes from ordinary households rather than from shops, factories or restaurants. Some of it was never edible: bones, peelings, eggshells, tea bags. Most of it was food that somebody bought, meant to eat, and did not.

Once it is in the bin, it can take several very different paths. It may go to landfill, where it rots without oxygen and releases methane. It may be burned in an energy-from-waste plant, although wet food is a poor fuel. It may go to an anaerobic digester, where microbes convert it into biogas that generates electricity or is injected into the gas grid, leaving a nutrient-rich digestate that returns to farmland. It may be composted. A small fraction is redistributed to people through charities and apps before it ever becomes waste. And a growing area of research asks whether wasted food should be treated not as rubbish at all, but as a raw material from which animal feed, protein, bioplastics or industrial chemicals can be made.

Which path the food actually takes depends on where you live and on what year it is. The rules in England changed on 31 March 2026, when councils became legally required to offer every household a separate weekly food waste collection. Wales has done something similar for years and now has one of the highest recycling rates in the

world, while Scotland has banned biodegradable waste from landfill altogether. Engineers and environmental scientists usually organise their thinking about all of this using the waste hierarchy, which ranks prevention above redistribution, redistribution above animal feed, animal feed above energy and nutrient recovery, and everything above landfill. This is an unfinished story, and one you can follow as it happens.

Guidance for your investigation tasks and research questions

You may find it easiest to begin by mapping out all the possible destinations for a piece of wasted food, then choosing one or two to explore in depth.

- How much food does the UK actually waste, and how does anyone know? Look into how organisations such as WRAP produce their estimates, using bin analyses, kitchen diaries and surveys, and try to work out why published figures disagree with one another.
- Choose one treatment route and explain how it works. What happens inside an anaerobic digester, an energy-from-waste plant or a landfill site? What goes in, what comes out, and how much useful energy or fertiliser is recovered?
- How do researchers compare these routes against one another, and what do the comparisons conclude? Which assumptions have the largest influence on the answer?
- Why is it illegal in the UK to feed most food waste to pigs, when Japan and South Korea do exactly that on a large scale? What was the reasoning behind the ban, and what do researchers argue today?
- Prevention sits at the top of the waste hierarchy, so why does so much of the debate concentrate on treatment instead? Why do households waste food, and which interventions have actually been shown to work?
- What is the current research on converting food waste into higher-value products such as bioplastics, proteins or chemicals, and is that genuinely better than simply recovering the energy?

If you would like to gather some evidence of your own, you could run a food waste audit for a week at home or in the school canteen, weighing and sorting what is thrown away and recording why, then scale your results up and compare them with the national figures. Finding out where your own council actually sends its food waste, and what happens to it there, would also make a strong addition.

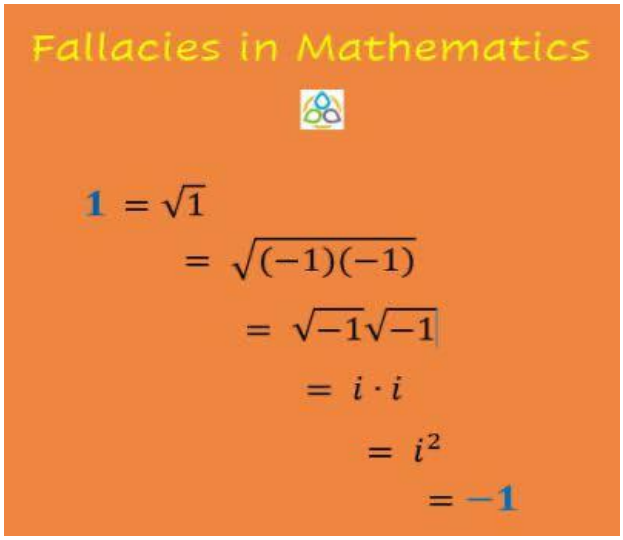
Project output

Your project output should comprise a literature review and a presentation, focused on a particular theme within the story of UK food waste rather than trying to cover all

of it. A summary of any small-scale investigation of your own, together with a discussion of what you found, would be a valuable addition. Useful starting points include WRAP, Defra and the GOV.UK pages on the Simpler Recycling reforms.

Project 17: How to Break Mathematics (And How to Fix It)

Supervisor: Jonathan Deane, Mathematics Department, University of Surrey



Project context

In school mathematics, we spend most of our time learning how to use algebraic rules, factorise equations, and differentiate functions to find solutions. Because these methods work reliably in class, it is easy to assume that formulas are universal and cannot lead to contradictions.

However, when rules are applied mechanically without checking their hidden conditions, mathematics breaks down completely—allowing us to "prove" absurdities such as 1 = 2 or 0 = 1.

For example:

- Suppose $a = b \neq 0$
  - $\Rightarrow a^2 = aa = ab$
  - $\Rightarrow a^2 - b^2 = ab - b^2$ (Subtracting b^2 from both sides)
  - $\Rightarrow (a-b)(a+b) = b(a-b)$ (Factorising)
  - $\Rightarrow a+b = b$ (Cancelling common factor)
  - $\Rightarrow 2b = b$ (Since $a = b$)
  - $\Rightarrow 2 = 1$

This project investigates the exact failure points of classic mathematical fallacies across algebra, sequences, and introductory calculus, demonstrating why operational rules require strict boundaries and definitions.

Core Research Question & Areas of Investigation

Overarching Question: Where do intuitive mathematical rules fail, and why are strict definitions necessary to prevent contradictions? Working within an accessible sixth-form scope, you will investigate:

• **Algebraic Disguises & Division by Zero:** Dissecting classic proofs of 1 = 2 and indeterminate expressions like 0 / 0 = 2. Students will explore how factor cancellation hides division by zero, why division by zero destroys the uniqueness of solutions, and how these illegal steps can be disguised in geometry or quadratic equations.

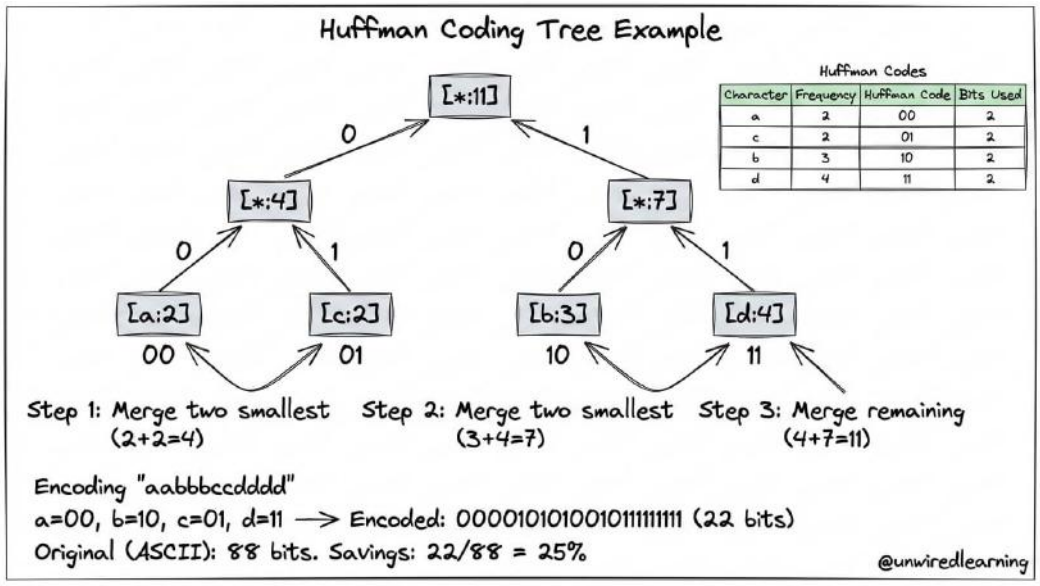
- **The Traps of Infinity & Grouping Rules:** Exploring why finite arithmetic rules fail for infinite series, such as Grandi's series ($1 - 1+1-1+\cdots = 0$ or 1) and geometric progressions ($1+2+4+8+\cdots = -1$). Students will investigate how shifting brackets violates associativity and evaluate how the definition of partial sum limits resolves the paradox.
- **Hidden Conditions in Calculus:** Investigating fallacies that arise in differentiation, such as expanding x^2 into an x-term sum ($x+x+\cdots+x$) to conclude that 2 = 1. Students will analyse why differentiation requires continuous domains rather than discrete whole numbers, and how variable term counts break the standard sum rule.

Starting Steps & Potential Extensions

- **Initial Steps:** Select 2–3 fallacies from algebra, infinite series, or basic calculus. Identify the exact line where an assumption is broken, test numerical values to watch the step fail, and write out the formal condition that prevents it.
- **Potential Extension** — Complex Fallacies & Branch Cuts: Active students can push beyond school maths to investigate why radical rules fail over complex numbers, for example:
    - $1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = (i)(i) = i^2 = -1$
    - $\Rightarrow -1=1$, or how multivalued complex logarithms lead to false proofs that $2\pi i = 0$.
- **Potential Extension** — **Visual & Geometric Paradoxes**: Students can investigate classic visual fallacies where geometric intuition breaks down. Topics could include the "$\pi = 4$ staircase paradox", the classical proof that "all triangles are isosceles", or the coin rotation paradox

Project 18: How Short Can We Make This Project? An Exploration of Compression, Coding and Entropy

Supervisor: Tom Hebdige, School of Engineering, University of Surrey



Project context

How efficiently can we send and store information? How much can we compress data? This is clearly an important question in our digital world, and the answer underpins compression algorithms used in common file types such as GIF and JPEG. In this project, we will explore several different compression algorithms. Furthermore, we will see how the probability distribution underlying the data affects how much we can compress data. Finally, we will study entropy, and see how this provides a fundamental limit on compression rates.

Literature review

- History of compression and encoding
- Exploration of letter frequency distributions
- Specific coding schemes, like Huffman coding or Lempel-Ziv schemes
- Entropy and limits of compression

Main project

Initial explorations:

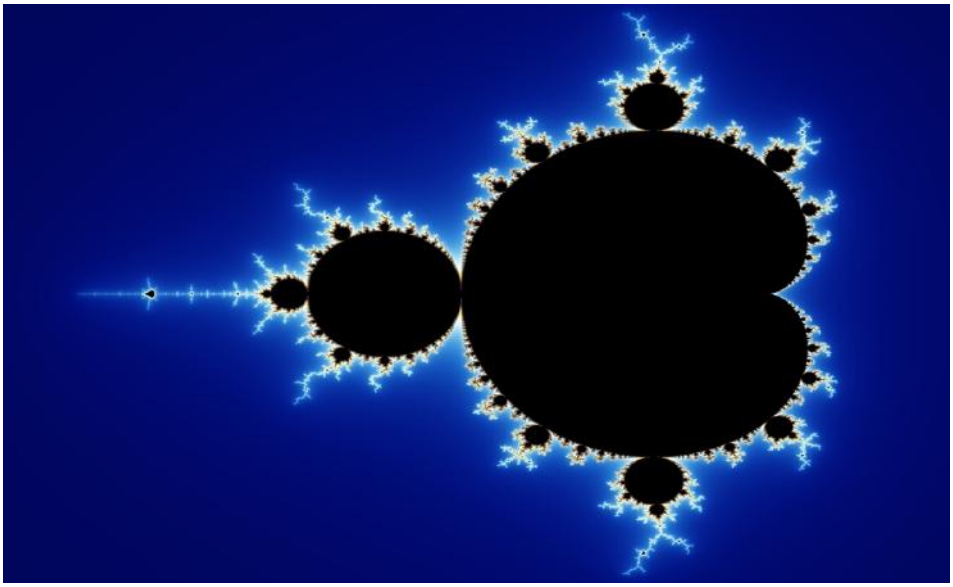
- Understand how underlying probability distribution affects efficiency of encoding
- Use the letter frequency distribution to find optimal coding rate
- Implement a Huffman coding scheme using the letter frequency distribution

Possible further exploration:

- Compare other encoding schemes e.g. Lempel-Ziv
- Consider encoding with multiple letters (i.e. consider distribution of letter pairs)
- Implement encoding schemes in Python (or other language)

Project 19: Exploring Fractals: Sequences and the Mandelbrot Set

Supervisor: Jan Gutowski, Department of Mathematics, University of Surrey



Introduction

Dynamical systems appear throughout nature, from understanding planetary orbits, to the weather, as well as in population modelling. Predicting the future using mathematical equations is of critical importance. In the 20th century, the introduction of computers provided a very direct way to visualize the remarkable insight that even simple equations can have complicated dynamics. This project introduces the strange world of fractals. The first part of the project involves researching key mathematical ideas relating to complex numbers, and some basic properties of sequences. The later parts of the project involve some numerical calculations, to produce an approximation of the Mandelbrot set, as well as investigating formal mathematical proofs which are used to understand key properties of the Mandelbrot set. We can also examine the relationship between the Mandelbrot set and the logistic sequence, which is related to population modelling.

Questions to investigate

Initial background research

- Explore the properties of complex numbers, covering: definitions, addition, multiplication and modulus of complex numbers.
- Investigate the geometric interpretation of complex numbers via the Argand diagram, and the triangle inequality for complex numbers.
- Investigate some simple sequences (e.g. the Fibonacci sequence) using induction.

Investigate the Mandelbrot set

- The following sequence of complex numbers is used to generate the Mandelbrot set:

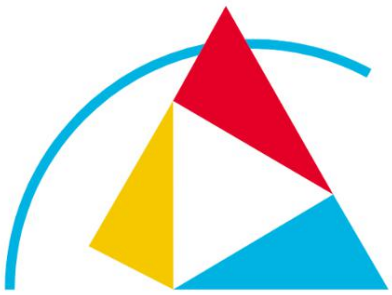
$$z_{n+1} + 1 = z_n^2 + c$$

where c is a complex constant. Depending on the choice of c, this sequence may converge, or diverge, or remain bounded without either converging or diverging. The Mandelbrot set consists of those choices of c such that the sequence stays bounded.

- Using numerical calculations, produce an approximation to the Mandelbrot set.
- Prove more formally (using mathematical induction) some results which can be used to show that a particular value of c is not an element of the Mandelbrot set and investigate some values of c for which the sequence converges.

Extension questions

- Investigate geometric properties of the Mandelbrot set.
- Investigate the logistic sequence – its relationship to the Mandelbrot set and its properties.



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